Einstein's Math Error in Special Relativity

8. Transformation of Velocity (Calculations from Eisenberg- Fundamentals of Modern Physics; pages 27-29)

Consider the particle shown in figure (1-10), moving with velocity ***v*** as seen in a frame of reference O, We would like to evaluate the velocity ***v'*** of the particle, as seen in the frame of reference *0'* which is itself moving relative to *0* with velocity ***v***.

Measured in the *0* frame, the velocity vector of the particle has components

$$Vx = \frac{dx}{dt} \ldots\ldots\ldots\ldots Vy = \frac{dy}{dt} \ldots\ldots\ldots\ldots\ldots\ldots Vz = \frac{dz}{dt}$$

The same velocity vector, as measured in the *0'* frame, has components

$$V'x = \frac{dx'}{dy'} \ldots\ldots\ldots\ldots V'y = \frac{dy'}{dt'} \ldots\ldots\ldots\ldots\ldots V'z = \frac{dz'}{dt'}$$

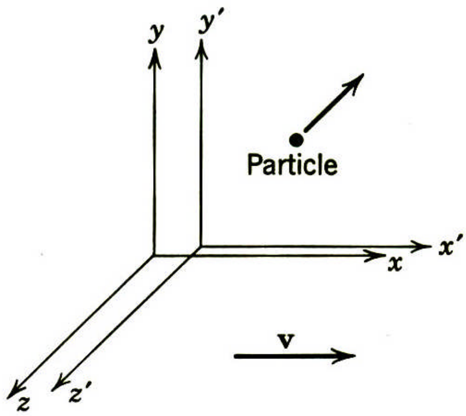

Figure 1-10. A moving particle observed from two frames of reference in uniform translation.

Now from equations (1-13) we know that the relation between the primed and the unprimed coordinates and times is

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Take the differential of these equations, remembering that **v** is a constant. This gives

$$dx' = \frac{dx - vdt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$dy' = dy$$

$$dz' = dz$$

$$dt' = \frac{dt - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$V'x = \frac{dx'}{dt'} = \frac{\frac{dx - vdt}{\sqrt{1-\frac{v^2}{c^2}}}}{\frac{dt - \frac{vdx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{dx - vdt}{dt - \frac{vdx}{c^2}} = \frac{\frac{1}{dt}(dx - vdt)}{\frac{1}{dt}(dt - \frac{vdx}{c^2})} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{Vx - v}{1 - \frac{v}{c^2}Vx} \qquad (1\text{-}16)$$

$$\ldots\ldots\ldots\ldots\ldots V'x = \frac{Vx - v}{1 - \frac{v}{c^2}Vx}$$

$V'x = coordinate..of..particle..relative..to..moving(primed)..system$

$Vx = coordinate..of..particle..relative..to..stationary..system$

$v = velocity..of..primed..coordinate..system..relative..to..stationary..system$

According to Einstein, this equation tells you how to transform the observed velocity from one frame of reference to another frame of reference.

First we note that, as V/c and *v*/c approach zero, the equation approaches those which would be derived from the Galilean transformation.

$$V'x = \frac{Vx - v}{1 - \frac{v}{c^2}Vx} = \frac{Vx - v}{1 - \frac{(v \to c)}{c^2}(Vx \to c)} = \frac{Vx - v}{1 - 0} = Vx - v$$

Another property of this equation is that it is impossible to choose V and v such that V', the magnitude of the velocity which is seen in the new frame, is greater than c.

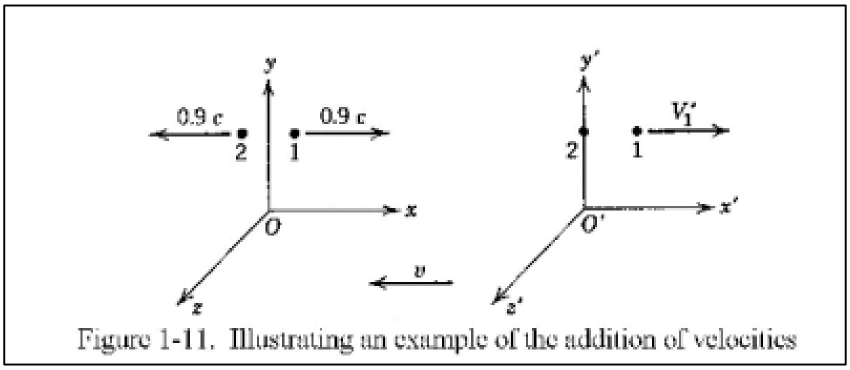

Figure 1-11: Illustrating an example of the addition of velocities.

Consider the example illustrated in figure 1-11. As seen by O, particle 1 has velocity 0.9c in the direction of positive x, and particle 2 has velocity 0.9c in the negative x direction. To evaluate the velocity of particle 1 with respect to particle 2 we transform from the O frame to the O' frame moving in the negative x direction with velocity v = -0.9c, using equation 1-16. We obtain:

$$V_1' = \frac{0.9c-(-0.9c)}{1-\frac{(-0.9c)}{c^2}(0.9c)} = \frac{1.80c}{1.81} < c = 0.9876543c$$

BS math caused Einstein's incorrect answer

$$V_1' = \frac{0.9c \overset{subtraction}{-} (\overset{direction-left}{-} 0.9c)}{1 \overset{subtraction}{-} \frac{(\overset{direction-left}{-} 0.9c)}{c^2} \overset{direction-right}{(0.9c)}} = \frac{1.80c}{1.81} < c = 0.9876543c$$

$$V_1' = \frac{\overrightarrow{0.9c} \overbrace{+\overrightarrow{0.9c}}^{sub.of.a.dir.to.the.left.is.addition.to.the.right}}{1 \overset{sub}{-} \frac{(\overleftarrow{0.9})(\overrightarrow{0.9})c^2}{c^2}} = \frac{1.8c}{1-\underbrace{(\overleftarrow{0.9})(\overrightarrow{0.9})}_{cannot.multiply.a.left.times.a.right}} = ?$$

SM shows the math error

SM:

- Cannot multiply a subtraction operator times a direction in space.
  - Can only solve for the resultant.
- Cannot multiply a direction to the left times a direction to the right.
  - Cannot multiply opposite directions in space.
  - Can only solve for the resultant

SM logical math produces a common sense answer

$$V_1^{'} = \frac{\overbrace{\overrightarrow{0.9c}} \overset{subtraction}{-} \overbrace{\overleftarrow{0.9c}}}{1 \overset{subtraction}{-} \frac{\overbrace{\overleftarrow{0.9c}} \& \overbrace{\overrightarrow{0.9c}}}{c^2}} =$$

sub of a dir to the left is a dir to the right

$$V_1^{'} = \frac{\overrightarrow{0.9}c \quad \overbrace{+\overrightarrow{0.9}c}}{1 \overbrace{-}^{sub} \frac{(\overbrace{\overleftarrow{0.9}}^{left}) \& (\overbrace{\overrightarrow{0.9}}^{right}) c^2}{c^2}} = \frac{1.8c}{1 \overbrace{-}^{sub} \frac{(0)c^2}{c^2}} = \frac{1.8c}{1} = 1.8c$$