Motion of Mass in Gases (Old Kinetic Energy)

All matter is in motion relative to other mass. There is no known mass that is not in motion relative to other mass. $(M\vec{a}d)$: $\overset{kg}{mass}\ \overbrace{accelerated}^{m/s^2}\ \overbrace{\text{distance}}^{meters}$

½ the average $(M\vec{a}d)$ of a gas molecule is $[(3/2)kT]$: $\left[\overset{1/2average}{M\vec{a}d} = (3/2)kT_A\right]$

- (T_A) is the absolute temperature of the gas
- $(k = 1.381 < 23(kg)(m/s^2)(m)/T)$ is Boltzmann's constant

For a molecule of mass (M), ½ the average $(M\vec{a}d)$ is: $\overbrace{(1/2)(M\vec{v}^2)}^{1/2average(M\vec{a}d)} = (3/2)kT_A$

The "Root Mean Square" velocity is: $\vec{v}_{rms} = \sqrt{3kT_A/M}$

Avogadro's Number:

$(\#_A) = 6.022141 > 26\,atoms\ or\ molecules/kmol = 6.022141 > 23a\ or\ m/mol$

P&N	Gas	#Avogadro	grams	grams/mol	*M/V* - STP
		B	C	B*C	kg/m^3
1P	H - Hydrogen	6.022141E+23	1.673725E-24	1.00794	
2P	H2 - H molecule	6.022141E+23	3.347452E-24	2.01588	0.09
2P2N	He - Helium	6.022141E+23	6.646482E-24	4.00260	0.166
7P7N	N - Nitrogen	6.022141E+23	2.325875E-23	14.00675	
14P14N	N2 - Nitrogen molecule	6.022141E+23	4.651751E-23	28.01350	
8P8n	O - Oxygen	6.022141E+23	2.656765E-23	15.99941	
16P16N	O2 - Oxygen molecule	6.022141E+23	5.313529E-23	31.99882	1.429
9P10N	F - Fluorine	6.022141E+23	3.154761E-23	18.99842	
10P10N	Ne - Neon	6.022141E+23	3.350837E-23	20.17921	0.899
17P20N	Cl - Chlorine	6.022141E+23	5.887063E-23	35.45273	
18P22N	Ar - Argon	6.022141E+23	6.633526E-23	39.94803	
	Kr - Krypton	6.022141E+23	1.391533E-22	83.80006	
	Xe - Xenon	6.022141E+23	2.180123E-22	131.29009	
	Rn - Radon	6.022141E+23	3.686399E-22	222.00016	
		6.022141E+23			
		6.022141E+23			

$$(M = \frac{W_{atomic}}{\#_A} = \frac{kg/kmol}{6.022141 > 26\, particles/kmol}) \qquad (1.1)$$

$$[M = (0.166 < 26kg)(W) = (0.166 < 23g)(W)]$$

The $\left[P_{pressure} = \frac{M\vec{a}}{A_{area}} \quad units\ are\ \frac{(kg)(m/s^2)}{m^2}) \right]$ of an ideal gas has been shown to be:

$$\left[PV = (1/3)(\#_{molecules})(M\vec{v}^2_{rms}) \right] \qquad (1.2)$$

Where $(\#_{molecules})$ is the number of molecules in the volume (V).

Dividing by (V) yields $(D_{density})$:

$$\left[P = (1/3)(\#_{molecules})(M/V)\vec{v}^2_{rms}) = (1/3)(\#_{molecules})(D_{density})(\vec{v}^2_{rms}) \right]$$

$$P = (1/3)(\#_{molecules})(D)(\vec{v}^2_{rms}) \qquad (1.3)$$

The Absolute Temperature of and ideal gas is a measure of its $(M\vec{a}d)$ per molecule

$$\left[\begin{array}{l} (1/2)(M\vec{v}^2) = (3/2)kT \quad \therefore \quad M\vec{v}^2 = 3kT \\ T = \frac{M\vec{v}^2_{rms}}{3k} = \frac{M\vec{v}^2_{rms}}{(3)(1.381 < 23)} = 2.4137 > 22(M\vec{v}^2_{rms}) \end{array} \right]$$

$$T = 2.4137 > 22(M\vec{v}^2_{rms}) \qquad (1.4)$$

The relation between:

$$\left[(k = 1.381 > 23\frac{(kg)(m/s^2)(m)}{(T)}), (R = 8314\frac{(kg)(m/s^2)(m)}{(kmol)(T)}), (\#_A = 6.0222 > 26\frac{particles}{kmol}) \right]$$

is: $\left[k = \frac{R}{\#_A} = \frac{8314\frac{(kg)(m/s^2)(m)}{(kmol)(T)}}{6.0222 > 26\frac{particles}{kmol}} = 1.381 < 23\frac{(kg)(m/s^2)(m)}{T} \right]$ Boltzmann's constant

The MEAN FREE PATH of a gas molecule is the average distance the molecule moves between collisions. Using a spherical molecule with radius (r) to make the math easier (JK: molecules are not spherical), a good estimate of the MEAN FREE PATH for an ideal gas is:

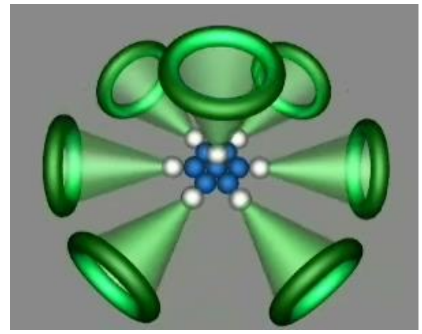
$$\left[\begin{array}{l} MFP = \frac{1}{(4\pi)(\sqrt{2})}\frac{1}{(\#/V)(r^2)} = (0.05627)\frac{1}{(\#/V)(r^2)} = (0.05627)\frac{V}{(\#_A)(r^2)} \\ MFP = (0.05627)\frac{V}{(\#_A)(r^2)} \end{array} \right]$$

Where $(\#/V)$ is the number of molecules per unit volume.

Problems:

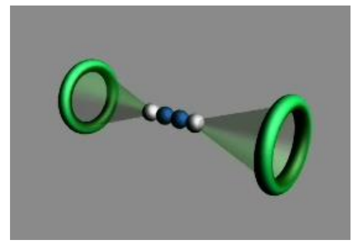
1: Find the mass (M) of a nitrogen molecule (N_2).

The molecular weight is $(W = 28\,kg/kmol)$:
$(7e7P7N)$



A: $$(M = \frac{W}{\#_A} = \frac{28kg\,/\,kmol}{6.022 > 26\,\frac{particles}{kmol}} = 4.65 < 26kg)$$

2: How many helium atoms (He) are there in $(2.0grams = 0.002kg)$ of helium? $[(W = 4\,kg/kmol)\ for\ He]$
$(2e2P2N)$



A: METHOD-1: Changing $(grams\ to\ kmol)$:

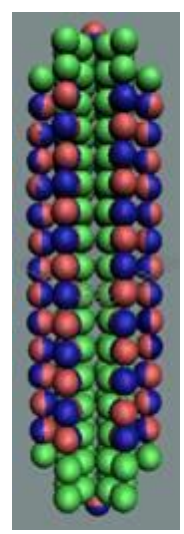
$$\left[\frac{M}{W} = \frac{0.002kg}{4\,kg/kmol} = 0.0005kmol\right]$$

$$\left[\begin{aligned}&\#\ of\ atoms\ in\ 2grams = (0.0005kmol)(\#_A)\\ &= (0.0005kmol)(6.022141 > 26\,atoms/kmol) = 3.01 > 23atoms\end{aligned}\right]$$

METHOD-2:

$$\left[\begin{aligned}&(M = \frac{W}{\#_A} = \frac{4kg\,/\,kmol}{6.022141 > 26atoms\,/\,kmol} = 6.64 > 27kg)\\ &in\ 2\ grams: \quad \frac{0.002kg}{6.64 > 27kg} = 3.01 > 23atoms\end{aligned}\right]$$

3: A droplet of mercury (Hg) has a radius of $(0.5 < 3m)$. How many mercury atoms are in the droplet?
$[(W = 202\,kg/kmol) \quad (D = M/V = 13600\,kg/m^3)]$
Because mercury is not spherical, an estimate will be:



Volume of droplet: $[V = (4\pi/3)(r^3) = (4.1879)(5 < 4m)^3 = 5.24 < 10m^3]$

Mass of droplet: $[M = (M/V)(V) = (13600\,kg/m^3)(5.24 > 10m^3)7.1 > 6kg]$

Mass of atom: $$\left[M = \frac{W}{\#_A} =\right] \frac{202\,kg/kmol}{6.022141 > 26\,atoms/kmol} = 3.36 < 25kg$$

\# of atoms in droplet: $$\left[\# = \frac{M_{droplet}}{M_{atom}} = \frac{7.1 < 6kg}{3.36 < 23kg} = 2.1 > 19atoms\right]$$

An image of the benzene cluster

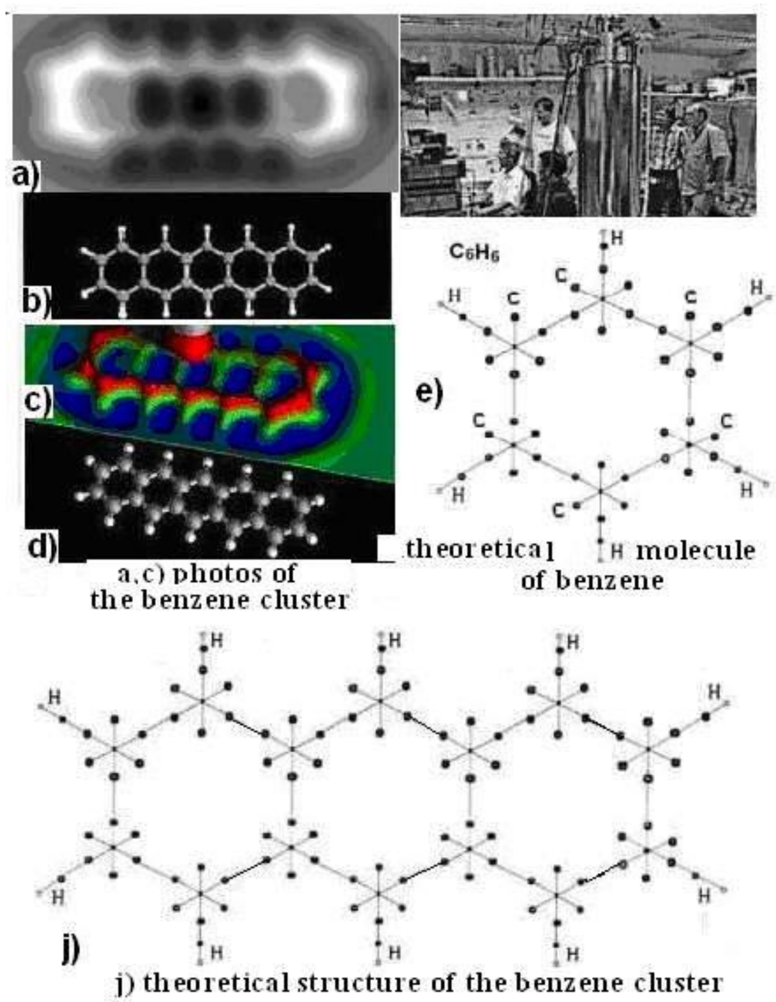
$(N_6 H_6)$

(a, c) photos of the benzene cluster;

(b, d) computer processing of the benzene clusters;

(e) theoretical molecule of benzene $\tilde{N}_6 H_6$;

(j) theoretical structure of the benzene cluster



Molar mass: $(W = 78\, kg/kmole)$

$$(D = \frac{M}{V} = 880 \frac{kg}{m^3})$$

4: How many molecules are there in $(70cm^3 = 70 < 6m^3)$ of benzene?

$(W = 78\, kg/kmol)$ $(D = M/V = 880\, kg/m^3)$

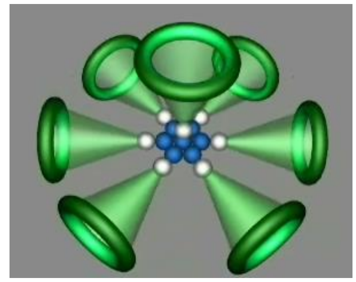
A: Mass of $(70cm^3 = 70 < 6m^3)$ $\left[M = (M/V)(V) = (880\, kg/m^3)(70 < 6m^3) = 0.0616 kg \right]$

Mass of molecule: $M = \frac{W}{\#_A} = \frac{78\, kg/kmol}{6.022141 > 26\, molcules/kmol} = 1.30 < 25kg$

# of molecules in $(70cm^3 = 70 < 6m^3)$:

$$\left[\#_{molecules} = \frac{M_{70cm^3}}{M_{molecule}} = \frac{0.0616kg}{1.30 < 25kg} = 4.8 > 23 molecules \right]$$

5: Find the $(\vec{v}_{rms})$ of a nitrogen molecule $(W = 28\,kg/kmol)$ in air at $(273.15^o = 0^o C)$. $(7e7P7N)$



A: $$\left[M = \frac{W}{\#_A} = \frac{28kg\,/\,kmol}{6.022141 > 26\,particles\,/\,kmol} = 4.65 < 26kg \right]$$

$$\left[\vec{v}_{rms} = \sqrt{3kT/M} = \sqrt{\frac{(3)(1.38<23\left[(kg)(m/s^2)(m)/T\right](273.15^o T)}{4.65<26kg}} = 490\,m/s \right]$$

6: If a gas molecule were to go straight up from the surface of the earth without colliding with other molecules, how high would it rise?

A: $\left[\overbrace{(1/2)(M\vec{v}^2)}^{average(M\vec{a}d)} = (3/2)kT \right]$ It is the average $(M\vec{a}d)$ that supplies the momentum exchange to move the gas molecule.

Gravitational $(M\vec{a}/A)$ is $(M\vec{a}_g h)$.

$$\left[\begin{array}{l} 3/2kT = M\vec{a}_g h \\ h = (\frac{1}{M})(\frac{(3)(1.38<23\left[(kg)(m/s^2)(m)/T\right](273.15^o T)}{(2)(9.8\,m/s^2)} = \frac{5.8<22kgm}{M} \end{array} \right]$$

The height is inversely proportional to the (M) of the molecule.

For the nitrogen molecule in the last problem, $\left[h = \frac{5.8<22kgm}{4.65<26kg} = 12.4km \right]$

7: Find the ratio of hydrogen $(W = 2\,kg/kmol)$ and nitrogen $(W = 28\,kg/kmol)$ gases at the same temperature.

A: (a) $\left[\dfrac{M\vec{a}d_H}{M\vec{a}d_N}\right]$ (b) $\left[\dfrac{\vec{v}_{H-rms}}{\vec{v}_{N-rms}}\right]$

The average translational $(M\vec{a}d)$ of a molecule is $(3/2\,kT)$ depends only on temperature.

$$\left[\frac{\vec{v}_{H-rms}}{\vec{v}_{N-rms}} = \sqrt{\frac{3kT/M_H}{3kT/M_N}} = \sqrt{\frac{M_N}{M_H}} = \sqrt{\frac{28}{2}} = 3.74\right]$$

8: Treating certain ideal gas molecules as spheres of radius $(r = 3<10m)$, find the MFP of these molecules under STP.

Method 1

A: At STP, one kilomol of substance occupies $(22.4m^3)$

$$\left[\begin{array}{l}(MFP) = \dfrac{1}{(4\pi)(\sqrt{2})}\dfrac{1}{(\#/V)(r^2)} = (0.05627)\dfrac{1}{(\#/V)(r^2)} = (0.05627)\dfrac{V}{(\#_A)(r^2)} \\ = (0.5627)(\dfrac{22.4m^3}{(6.022141>26 molecules/mol)(3>10m)^2} = 2.4<8m\end{array}\right]$$

Method 2

$$\left[(M\vec{a}/A)(V) = \#kT \quad \therefore \quad \frac{\#}{V} = \frac{M\vec{a}/A}{kT} = \frac{1>5(kg)(m/s^2)/m^2}{(1.38<23\left[(kg)(m/s^2)(m)/T\right](273^oT)} = \frac{2.65>25}{m^3}\right]$$

$$\left[(0.05627)\frac{1}{(\#/V)(r^2)} = (0.05627)\frac{1}{(\frac{2.65>25}{m^3})(3<10m^2)} = 2.4<8m\right]$$

9: At what $(P = M\vec{a}/A)$ will the MFP be $(50cm = 0.5m)$ for spherical molecules of radius $(r = 3 < 10m)$ and a temperature of $(293.15^o T = 20^o C)$?

A. Rearranging : $\left[\frac{\#}{V} = (0.05627)\frac{1}{(\#/V)(r^2)}\right]$ and $\left[\frac{\#}{V} = \frac{M\vec{a}/A}{kT}\right]$,

we have:

$$\left[(P = M\vec{a}/A) = (0.05627)\frac{kT}{r^2(MFP)} = \frac{(1.38 < 23)(293^o)}{(3 > 10m^2)(0.5m)} = 5.1 < 3(kg)(m/s^2)/m^2\right]$$