IDEAL GASES (JK Physics-Heat & Temp- Ideal Gases)

Ideal Gas: An ideal gas is one that obeys the equation

$$\left[PV = \overbrace{\left(\frac{M\vec{a}}{m^2} \right)}^{\text{Pressure}} \overbrace{(m^3)}^{Volume} = \left(\frac{M}{(\#_A)(M)} \right)(R)(T_A) = \left(\frac{M}{W} \right)(R)(T_A) \right] \quad (1.1)$$

$$\left[P = \left(\frac{M}{V} \right)\left(\frac{R}{W} \right)(T_A) = (D)\left(\frac{R}{W} \right)(T_A) \right] \quad (1.2)$$

- $P_{\text{Pressure}} = \frac{M_{mass}\vec{a}_{accleration}}{A_{area}} = \frac{M\vec{a}}{m^2}$
- $V_{Volume} = m^3_{meters}$
- $\begin{bmatrix} \#_A = Avogadro's\ number \\ = 6.022141 > 23 (atoms\ or\ molecules)/mol \\ = 6.022141 > 26 (atoms\ or\ molecules)/kmol \end{bmatrix}$
- $W_{atomic\ weight} = (\#_A)(M)$
- $\frac{M}{W} =$ is the number of $(kmol)$ in a volume (V)
- $R = 8314 \frac{(kg)(m/s^2)(m)}{(kmol)(T_A)} = 8.314 \frac{(g)(m/s^2)(m)}{(mol)(T_A)}$ Universal Gas Constant
- $T_A = Absolute\ \text{T}emperature = -273.15^o C$

Equation (1.1) holds for low to moderate pressure when the temperature is not too low.

Mol definition: A *mol* of a substance is the amount of that substance that contains as many particles as there are atoms in exactly 12 grams of carbon-12. It is more practical to use the kilomol (*kmol*). A *kmol* of substance is the mass (in *kg*) that is equal to the atomic weight of the substance. For example:

The atomic weight of hydrogen gas is 2. There are $(2kg)$ in $(1kmol)$ of H2.

- There is $(2grams)$ in $(1mol)$.

The atomic weight of oxygen gas is 32. There are $(32kg)$ in $(1kmol)$ of O2.

- There is $(32grams)$ in $(1mol)$.

$$\left[\text{For } \mathrm{H}_2: \ W_{atomic\ weight} = (\#_\mathrm{A})(\mathrm{M}) = (\overbrace{6.022141 > 23}^{Avogadro's\#} molecules)(3.34745 < 27kg) = 2.01588kmol \right]$$

$$\left[\text{For } \mathrm{H}_2: \ \left(\frac{M}{W_{atomic\ weight}} \right) R = \frac{3.34745 < 27kg}{2.01588kmol}(8314) = (1.66054 < 27)(8314) = \overbrace{1.381 < 23}^{Boltzmann's\ \text{constant}} \ \frac{kg}{kmol} \right]$$

Standard Temperature & Pressure (STP):

$$\begin{bmatrix} \text{Standard Temperature} & \\ T_A & = 273.15^o = 0^o C \end{bmatrix} \quad \begin{bmatrix} \text{Standard Pressure} & \\ P & = \dfrac{M\vec{a}}{m^2} = 1.013 > 5 \dfrac{(kg)(m/s^2)}{m^2} \end{bmatrix}.$$

Under (STP), $(1kmol)$ of an ideal gas occupies $(22.4m^3)$.

Under (STP), $(2kg\ of\ H_2,\ 32kg\ of\ O_2,\ \ 28kg\ of\ N_2)$ all occupy $(22.4m^3)$.

Changes from $(P_1,V_1,T_1\ \ to\ \ P_2,V_2,T_2)$: $\left[\dfrac{P_1V_1}{T_1} = \dfrac{M}{W}R = \dfrac{P_2V_2}{T_2}\right]$ for constant (M).

$$\left[\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}\right] \tag{1.3}$$

NOTE: (M) is never constant. When a photon is absorbed, the mass increases. When a photon is emitted, mass is decreased. The change is out to the 9^{th} digit, but there is a change in mass.

Gauge Pressure: Gauges read the excess of pressure over atmospheric pressure and this excess is called "gauge pressure". While a useful measurement for many practical purposes, it must be converted to absolute pressure for the ideal gas law.

$$\begin{bmatrix} (M\vec{a}/m^2)_{gauge} & = (M\vec{a}/m^2)_{absolute} & - (M\vec{a}/m^2)_{atmosphere} \\ P_{gauge} & = P_{absolute} & - P_{atmosphere} \end{bmatrix}$$


When a system is at atmospheric pressure like the left image above, the gauge pressure is said to be zero. In this image, the system has been opened so that it is at equilibrium with the atmosphere.

In the right image, the system has been closed and the plunger pushed down until the pressure is $(15lb/in^2 = 103.421172 > 3(kg)(m/s^2)/m^2)$. This implies that the absolute pressure has been approximately doubled by compressing the gas to half its volume (ideal gas law). Standard atmospheric pressure is $(14.7\,lb/in^2 = 101.352749 > 3(kg)(m/s^2)/m^2)$, so this must be added to the gauge pressure above to get the absolute pressure.

Problems:

1: A mass of oxygen occupies $(V_1 = 0.02m^3)$ at $\left[101 > 3(kg)(m/s^2)/m^2\right]$ and $(5^oC = 278^oT)$;

? Determine (V_2) if $\left[P_2 = M\vec{a}/m^2\right]$ is increased to $\left[108 > 3(kg)(m/s^2)/m^2\right]$ while the temperature changes to $(30^oC = 303^oT)$?

A: $$\left[V_2 = (V_1)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right) = (0.02m^3)\frac{\left[101 > 3(kg)(m/s^2)/m^2\right]}{\left[108 > 3(kg)(m/s^2)/m^2\right]}\left[\frac{303^oT}{278^oT}\right] = 0.0204m^3\right]$$

NOTE: The mass increases because photons are absorbed by the oxygen. It is a very very small increase, but the mass does increase.

2: When the atmospheric pressure is $\left[14.5lb/in^2 = 99.9738 > 3(kg)(m/s^2)/m^2\right]$, the gauge on a tank reads the pressure inside to be: $\left[130\,lb/in^2 = 896.316827 > 3(kg)(m/s^2)/m^2\right]$.

The gas tank has a temperature of $(9^oC = 282^oT)$.

? If the tank is heated to $(31^oC = 304^oT)$ by the sun, what will the gauge read?

A: $\left[P_2 = (P_1)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right)\right]$ there is no change in volume, $\left[(V_1 = V_2) \quad \therefore \quad \left(\frac{V_1}{V_2}\right) = 1\right]$

$$\left[P_2 = (P_1)\left(\frac{T_2}{T_1}\right)\right] \qquad \left[P_{absolute} = P_{gauge} + P_{atmosphere}\right]$$

$$\left[P_{1-abs} = \overbrace{896.316827 > 3}^{(kg)(m/s^2)/m^2\ gauge} + \overbrace{99.9738 > 3}^{(kg)(m/s^2)/m^2\ atm} = 996.290627 > 3(kg)(m/s^2)/m^2\right]$$

$$\left[P_2 = (P_1)\left(\frac{T_2}{T_1}\right) = (\overbrace{996.290627 > 3}^{(kg)(m/s^2)/m^2})\left(\frac{304^o}{282^o}\right) = 1074.201244 > 3(kg)(m/s^2)/m^2\right]$$

The gauge will read:

$$\left[\begin{array}{l} P_{gauge} = \quad P_{absolute} \quad - \quad P_{atmosphere} \\ P_{gauge} = \overbrace{1074.201244 > 3}^{absolute} - \overbrace{99.9738 > 3}^{atmosphere} = \overbrace{974.227444 > 3}^{gauge}(kg)(m/s^2)/m^2 \end{array}\right]$$

3. Gas at room temperature and $\left[P_1 = 98.6585526>3(kg)(m/s^2)/m^2\right]$ is confined to a cylinder by a piston. The piston is now pushed in so as to reduce the volume (V) to $(1/8 = 0.125)$ of its original value.

? After the gas temperature has returned to room temperature, what will be the gauge $[P_2]$ of the gas?

A: $\left[P_2 = (P_1)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)\right]$ In this problem, $(T_1 = T_2)$. Therefore $\left[P_2 = (P_1)\left(\frac{V_1}{V_2}\right)\right]$ and $(V_2 = 0.125V_1)$.

Inserting these values into the equation, we have:

$$\left[\begin{array}{l} P_2 = (P_1)\left(\frac{V_1}{V_2}\right) = \overbrace{98.6585526>3}^{(kg)(m/s^2)/m^2}\left(\frac{\cancel{V}_1}{0.125\cancel{V}_1}\right) = \overbrace{98.6585526>3}^{(kg)(m/s^2)/m^2}(8) \\ = 789.2684208 > 3(kg)(m/s^2)/m^2 \end{array}\right]$$

Gauge Pressure will be:

$$\left[\begin{array}{l} P_{absolute} \quad - \quad P_{atmosphere} \quad = \quad P_{gauge} \\ \overbrace{789.2684208 > 3}^{absolute} - \overbrace{98.6585526>3}^{atmosphere} = \overbrace{690.609682>3}^{gauge}(kg)(m/s^2)/m^2 \end{array}\right]$$

4. If $(1liter = 0.001m^3)$ of gas at $\left[P_1 = 101.3 > 3(kg)(m/s^2)/m^2\right]$ and $(-20^oC = 253^oT)$ is compressed to $(0.5liters = 0.0005m^3)$ with a temperature change to $(40^oC = 313^oT)$, what was the $[P_2]$ pressure used to compress the gas?

A: $\left[P_2 = (P_1)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (101.3 > 3)\left(\frac{0.001}{0.0005}\right)\left(\frac{313}{253}\right) = 250.64 > 3(kg)(m/s^2)/m^2\right]$

5. The $\left[P_1 = 28\,lb/in^2 = 193.1>3(kg)(m/s^2)/m^2\right]$ in a car tire at $(T_1 = 0^oC = 273^oT)$. As the car runs on the road, the temperature of the tire increases.

? What is the temperature (T_2) of the tire when the tire gauge reads $\left[P_2 = 35\,lb/in^2 = 241.3>3(kg)(m/s^2)/m^2\right]$?

Assume that the volume of the tire remains constant $(V_1 = V_2)$.

Atmospheric pressure is $(P_A = 14.7\,lb/in^2 = 101.4 > 3(kg)(m/s^2)/m^2)$

A:

$$\begin{bmatrix} P_{1gauge} + P_{atmosphere} = P_{1absolute} \\ = \overbrace{193.1>3}^{gauge} + \overbrace{101.4>3}^{atmosphere} = \overbrace{294.5 > 3}^{absolute}(kg)(m/s^2)/m^2 \end{bmatrix}$$

$$\begin{bmatrix} P_{2gauge} + P_{atmosphere} = P_{2absolute} \\ \overbrace{241.3>3}^{gauge} + \overbrace{101.4>3}^{atmosphere} = \overbrace{342.7 > 3}^{absolute}(kg)(m/s^2)/m^2 \end{bmatrix}$$

$$\left[T_2 = T_1\left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right) = (273^o)\left(\frac{342.7>3}{294.5>3}\right)(1) = 318^oT\right]$$

$$\left[T^2 = 318^oT - 273^oT = 40^oC\right]$$

6: A mass of hydrogen gas occupies $(V_1 = 370cm^3 = 0.00037m^3)$ at $(T_1 = 16^oC = 289^oT)$ and $(P_1 = (M\vec{a}/m^2)_1 = 150 > 3(kg)(m/s^2)/m^2)$.

? Find (V_2) at $(T_2 = -21^oC = 252^oT)$ and $(P_2 = 420 > 3(kg)(m/s^2)/m^2)$?

A:

$$\left[V_2 = (V_1)\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right) = (0.00037m^3)\frac{\left[150>3(kg)(m/s^2)/m^2\right]}{\left[420>3(kg)(m/s^2)/m^2\right]}\left[\frac{252^oT}{289^oT}\right] = 0.000115m^3\right]$$

7: A steel tank contains oxygen gas at $(T_1 = 32^oF = 0^oC = 273^oT)$ and a total $\left(P_1 = (M\vec{a}/m^2)_1 = 12atm = (12)(101.3>3) = 1215.6>3(kg)(m/s^2)/m^2\right)$.

? Determine the gas (P_2) when the tank is heated to $(T_1 = 212^oF = 100^oC = 373^oT)$?

A:

$$\begin{bmatrix} P_2 = (P_1)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (1215.6>3)(1)\left(\frac{373}{273}\right) = (1215.6>3)(1)(1.3663) \\ = 1660.9(kg)(m/s^2)/m^2 \end{bmatrix}$$

8: The density $\left[D_1 = \frac{M(kg)}{V(m^3)}\right]$ of nitrogen is $(1.25 kg/m^3)$ at STP $\left[T_1 = 273^o \text{ and } P_1 = 760 mmHg = 101.3 > 3(kg)(m/s^2)/m^2\right]$.

? Determine the density $\left[D_2 = \frac{M}{V}\right]$ of nitrogen at $\left[T_2 = 42^o C = 315^o T\right]$ and $(P_2 = 730 mmHg = 97.3 > 3(kg)(m/s^2)/m^2)$. $\left[(D) = \frac{PW}{RT_A}\right]$

A: $\left[\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}\right]$ substituting $\left[V = \frac{M}{D}\right]$, we have: $\left[\frac{P_1 \cancel{M}}{D_1T_1} = \frac{P_2 \cancel{M}}{D_2T_2}\right]$ The mass does not change and cancels in the equation; rearranging to solve for (P_2):

$$\left[D_2 = D_1\left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) = (1.25)\left(\frac{97.3 > 3}{101.3 > 3}\right)\left(\frac{273^o}{315^o}\right) = 1.04\frac{kg}{m^3}\right]$$

9: A $(V_1 = 3000 cm^3 = 0.003 m^3)$ tank contains oxygen at $\left(T_1 = 20^o C = 293^o T\right)$ and a gauge pressure of $\left[P_1 = (M\vec{a}/m^2)_1 = 25 > 5(kg)(m/s^2)/m^2\right]$.

? What is the mass of oxygen in the tank?

The molecular weight of oxygen is 32. $(W = (M)(\# A))$

Assume atmospheric pressure to be $(1 > 5(kg)(m/s^2)/m^2)$

A: $$\left[\begin{matrix} P_{gauge} + P_{atmosphere} = P_{absolute} \\ (25 > 5)+(1 > 5) = 26 > 5(kg)(m/s^2)/m^2 \end{matrix}\right]$$

From the gas equation, $(W = 32\, kg/kmol)$

$\left[PV = (\frac{M}{W})R(T)\right]$. solving for (M):

$$\left[M = \frac{WPV}{RT} = \frac{(32)(26 > 5)(0.003)}{(8314)(293^o)} = 0.102 kg\right]$$

10: ?: Determine the (V) occupied by $(4 grams = 0.004 kg)$ of oxygen $(W = 32\, kg/kmol)$ at $\left[T = 273^o \text{ and } P = 101.3 > 3(kg)(m/s^2)/m^2\right]$.

$\left[R = 8314(kg)(m/s_2)(m)/kmol\right]$

A: $$\left[V = \frac{MRT}{PW} = \frac{(0.004)(8314)(293)}{(101.3 > 3)(32)} = 0.0028 m^3\right]$$

11: A $(2mg = 2 < 6kg)$ droplet of liquid nitrogen is present in a $(V = 30cm^3 = 3 < 5m^3)$ tube as it is sealed off at very low temperature.

?: what will be the nitrogen pressure in the tube when it is warmed to $(T = 20^o C = 293^o)$?

$$[(W = 28 kg/kmole) \text{ for nitrogen}] \quad [R = 8314(kg)(m/s_2)(m)/kmol]$$

A: $$\left[P = \frac{MRT}{VW} = \frac{(2<6)(8314)(293)}{(3<5)(28)} = 5800(kg)(m/s^2)/m^2\right]$$

12: A tank of $(V = 590liters = 0.59m^3)$ contains oxygen at $(T = 20^o C = 293^o)$ and $\left[P = 5atm = 506.5 > 3(kg)(m/s^2)/m^2\right]$.

? Calculate the mass of oxygen in the tank.

$$(W = 32 kg/kmol) \quad [R = 8314(kg)(m/s_2)(m)/kmol]$$

A: $$\left[PV = \frac{MRT}{W}\right] \quad \left[M = \frac{PVW}{RT} = \frac{(506.5>3)(0.59)(32)}{(8314)(293)} = 3.9kg\right]$$

13: At $(T = 18^o C = 291^o)$ and $\left[P = 760mmHg = 101.325 > 3(kg)(m/s^2)/m^2\right]$, $(V = 1.29liters = 0.00129m^3)$ of an ideal gas “weighs” $(M = 2.71grams = 0.00271kg)$.

?: Calculate $(W_{atomic\ weight})$ the weight of the gas.

A: $$\left[PV = \frac{MRT}{W}\right] \quad \left[W = \frac{MRT}{PV} = \frac{(0.00271)(8314)(291)}{(101.325>3)(0.00129)} = 50.16\frac{kg}{kmol}\right]$$

14:?: Compute the (V) of $(M = 8grams = 0.008kg)$ of helium $(W = 4 kg/kmol)$ at $(T = 15^o C = 288^o)$ and $(P = 480mmHg = 63.995>3(kg)(m/s^2)/m^2)$.

A: $$\left[PV = \frac{MRT}{W}\right] \quad \left[V = \frac{MRT}{WP} = \frac{(0.008)(8314)(288^o)}{(4)(63.995>3)} = 0.075m^3\right]$$

15: Find the $(D = \frac{M}{V})$ of methane $(W = 16\,kg/kmol)$ at $(T = 20^oC = 293^o)$ at $(P = 5atm = 506.625 > 3(kg)(m/s^2)/m^2)$.

A: $\left[PV = \frac{MRT}{W}\right]$ $\left[D = \frac{M}{V} = \frac{PW}{RT} = \frac{(506.625 > 3)(16)}{(8314)(293)} = 3.32\frac{kg}{m^3}\right]$

16: In a gaseous mixture at $(T = 20^oC = 293^o)$, the partial pressures of the components are as follows:

$$\begin{bmatrix} hydrogen, & 200mmHg = 26.6644737 > 3 \\ carbon\ dioxide, & 150mmHg = 19.9983553 > 3 \\ methane, & 320mmHg = 42.6631579 > 3 \\ ethylene, & 105mmHg = 13.9988487 > 3 \end{bmatrix} \begin{bmatrix} W_{hydrogen} = 2 \quad kg/kmol \\ W_{CO_2} = 44 \ kg/kmol \\ W_{methane} = 16 \ kg/kmol \\ W_{ethylene} = 30 \ kg/kmol \end{bmatrix}$$

? What are:

a) the total pressureof the mixture and

b) the mass fraction of hydrogen?

A: a)

$$\begin{bmatrix} total\ pressure = sum\ of\ partial\ pressures \\ P_{total} = 26.6644737 > 3 + 9.9983553 > 3 + 42.6631579 > 3 + 13.9988487 > 3 \\ = 103.3 > 3(kg)(m/s^2)/m^2 \end{bmatrix}$$

b)

$$\left[M_{total} = (W_H P_H + W_{co_2}P_{co_2} + W_{methane}P_{methane} + W_{ethlyene}P_{ethlyene})\left(\frac{V}{RT}\right)\right]$$

$$\begin{bmatrix} \frac{M_{hydrogen}}{M_{total}} = \frac{W_H P_H}{W_{co_2}P_{co_2} + W_{methane}P_{methane} + W_{ethlyene}P_{ethlyene})} \\ = \frac{(2)(26.66 > 3)}{(2)(26.66 > 3) + (44)(19.99 > 3) + (16)(42.66 > 3\ ) + (30)(13.99 > 3)} \\ = 0.026 \end{bmatrix}$$